Heaven is a Space on Earth: Geometry in sacred spaces in early modern Europe and Japan

Antonia Karaisl

1. Introduction

The progress of mathematics in Early Modern Europe is closely entwined with the Scientific Revolution and often enough, practical problems precipitated mathematical advance in diverse fields, be that navigation, astronomy, optics, or mechanics[1]. *Wasan* (和算), the mathematical discourse that developed in Japan throughout the Edo period (1603-1867), provides a striking contrast. Originally catalysed by the drip-drop influx of ancient Chinese treatises, *wasan*'s origins playfully reflect the material reality of a merchant or farmer; its development, however, was not necessarily driven by practical considerations. At its peak, *wasan* prospered in splendid abstraction: as mathematics for mathematics' sake, an intellectual pursuit with a social dimension. This is not to say that there was no such thing as applied mathematics in Edo Japan. Yet quite apart from utilitarian considerations, a nation-wide network of mathematics scholars emerged that valued the discipline as an art, akin to calligraphy, tea ceremony or ikebana[2].

1 Wilbur R. Knorr *et al.*, *Mathematics*, in «Encyclopedia Britannica», 28 Jan. 2024, https://www.britannica.com/science/mathematics. Accessed 29 January 2024.

2 Consider e.g. the preface to the *Sanpo koren* (算法琉璉) (1836) where Tadayoshi Kobayashi defends the lack of practical motivation to his *sangaku* problems, in Nobuya Nakamura, Tetsuo Sekiguchi, *Sanpo-Koren & Kankai-No-Utsuwa. Problems and Solutions*, Nagano City, Kyoikushokan, 2014, pp. 42-43.

DOI 10.35948/DILEF/978-88-6032-827-4.02

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East Asian scholars first got a taste of Europe's mathematical tradition in the sixteenth century when Jesuit missionaries brought their astronomy and geometry to China and Japan. Yet whilst the Aristotelian-Ptolemaic cosmos experienced an afterlife on Japanese shores, Euclid, Europe's poster child, seems to have fallen flat. Even with the introduction of a Chinese translation of the work at a later stage, Japanese mathematicians did not seem to assign any value to it[3]. The reason could simply have been that Euclid's *Elements* did not offer anything conducive to the discourse inherited from Chinese predecessors; in fact, said Chinese predecessors seem to have reacted rather coldly to said Chinese translation of the work themselves[4].

There is, however, a faint analogy in practice linking *wasan* with the Jesuits' reception of Euclid in Portugal: the display of geometric themes and problems in religious spaces. In the Jesuit college of Coimbra, tiles with Euclidean theorems adorned select classroom walls; in an eighteenth century mathematics auditorium in the Jesuit college in Sant'Antão, a cycle of decorative tiles depict mathematical disciplines in allegorical form, with a chosen theorem by Archimedes embedded in an emblematic detail[5]. In Japan meanwhile, *wasan* gave rise to the medium of *sangaku* (算額), beautifully illustrated votive tablets containing mathematical problems in word and image. *Sangaku* were

3 Osamu Kota, *Western Mathematics on Japanese Soil – A History of Teaching and Learning of Mathematics in Modern Japan*, in «Advanced Studies in Pure Mathematics», vol. 79, 2018, pp. 337-346: p. 337.

4 Jean-Claude Martzloff, *Geometry in Chinese Mathematics*, in Helaine Selin (ed.), «Encyclopaedia of the History of Science, Technology, and Medicine in Non-Western Cultures», Dordrecht, Springer, 2008, pp. 1005-1011: p. 1009.

5 Henrique Leitão, Samuel Gessner, *Euclid in Tiles: The Mathematical Azulejos of the Jesuit College in Coimbra*, in «Mathematische Semesterberichte», vol. 61, No. 1, 2014, pp. 1-5; Henrique Leitão, Samuel Gessner, *Una tribus ratio: Ikonographie der Wissensvermittlung und Selbstdarstellung der Jesuiten im Mathematiksaal des Kollegs Santo Antão in Lissabon*, in «Mathematische Semesterberichte», vol. 62, 2015, pp.1-6. Photographs and more detailed descriptions of the mathematical tiles in Coimbra and Sant'Antão can be found in Carlota Simões, António Leal Duarte (eds.), *Azulejos Que Ensinam*, Coimbra, Universidade de Coimbra, 2007.

dedicated in temples and shrines throughout the country, ostensibly to thank the Gods for the successful solution of a problem[6]. The custom flourished throughout Japan's Edo period, only to founder in the first decades of the 20th century. Today, approximately 900 extant tablets are still kept in temples and shrines throughout the country.

Fig. 1 Sangaku from Yamatotakeru Shrine, Koriyama, Fukushima Prefecture (dedicated 1881)

Whilst print publications tended to be the domain of established scholars, *sangaku* provided a medium for figures of lesser prominence to take part in the conversation. Beyond voice and paper, therefore, they presented an alternative form of dissemination from a broader range of contributors to a wider audience, benefiting from the footfall of temples and shrines[7]. Each *sangaku* typically contains one or more illustrated problems providing question, answer, and technique, alongside the dedicatees' name and affiliation. The problem's full solution, however, is not revealed, challenging punters to find their own. *Sangaku* typically solicit the calculation of lengths, areas, or volumes of geometric figures. *Prima facie*, the illustrations provided on each tablet seem reminiscent of the diagrams familiar from Euclid's *Elements* – in fact, they occasion-

6 Sujatha Ramdorai, *Interview with Hidetoshi Fukagawa*, in «Asia Pacific Mathematics Newsletter», vol.3, No.4, 2013, pp. 30-36: see p. 32.

7 Annick Horiuchi, *Japanese Mathematics in the Edo Period (1600-1868)*, trans. by Silke Wimmer-Zagier, Basel, Springer, 2010, p. 21.

ally have been described as "Euclidean"[8]. A closer inspection of the techniques employed, however, reveals that classic trappings of Euclidean geometry (for example, the use of angles, parallelism or perpendicular lines) are mostly absent, nor do the tablets signal the existence of an axiomatic knowledge system. Generally speaking, *wasan*'s formulae seem to derive from complex equations, not deductive logic.

In spite of the close chronological range – the rise of *wasan* coincides with the tail end of the Jesuit mission in Japan – there is no reason to assume that an actual link exists between the mathematical tiles in Portugal and *sangaku* in Japan. Yet both cases, though oceans apart, hint at an elevated role reserved for mathematics within the realm of human thought – elevated enough, in fact, to merit a space within religious contexts, too. The following article takes this observation as the point of departure for a comparative review of the relationship between mathematical thinking and spirituality in Christian philosophy and in Japan respectively. The argument will first cover the links between Euclidean geometry and Christian cosmology in Jesuit discourse, to then investigate the role of mathematics within Japan's multi-faith landscape.

2.1. Euclid and the Jesuits: A Royal Road to Heaven?

Euclid was a stable component of the *Quadrivium*, the curriculum for mathematical sciences fundamental to the study of Liberal Arts in Europe's universities throughout the Middle Ages. The rise of printing technology in the fifteenth century helped to release geometry's power beyond curricular confines, as the *Elements* were republished in translations old and new. Euclid's subsequent spread on the European continent sparked a quiet Renaissance of his work, with the result that «it crossed every intellectual, religious, linguistic, and disciplinary

8 Sujatha Ramdorai, *Interview with Hidetoshi Fukagawa*, cit., p. 34.

boundary», providing a «common language of space»[9]. The conceptual power of Euclid's *Elements* not only appealed to science in theory and practice, but inspired epistemological admiration from religious camps besides, judging from the Protestant scholar Philipp Melanchthon's preface to a 1536 edition of the *Elements*:

> No one without some knowledge sees enough of this art, which is life demonstrated. No one without it will be a maker of method [...]. There is here great praise of geometry, which did not cling to inadequate and inferior [human] constructions, but flew into heaven and transported human minds, which were stuck in the mud, back up to the heavenly throne[10].

Here, geometry is felt to exceed the boundaries of earth-bound science, providing glimpses of a higher-level order in the universe. This sentiment not only struck home with the Protestants but found a particular form of expression within the science and pedagogy promoted by the Jesuits. Founded in 1534, the Jesuit Order established a network of educational institutions across the world to support their proselytization efforts. Considerable care was invested in compiling one curriculum according to Humanist principles, the *Ratio Studiorum*, which was published in 1599 in an effort to ensure pedagogical consistency and intellectual rigour throughout Jesuit schooling[11]. Parts of Euclid's *Elements* were taught to students of natural philosophy, presumably thanks to the lobbying efforts of Christopher Clavius, eminent mathematician and astronomer based at the Jesuit College in Rome[12].

9 Michael J. Sauter, *For the Love of Geometry. The Rise of Euclidism in the Early Modern World, 1450-1850*, in Jason Coy, Benjamin Marschke, Jared Poley, Claudia Verhoeven (eds.), *Kinship, Community and Self*, New York, Berghahn Books, 2015, pp. 185-201: see pp. 185-186.

10 Quoted in Michael J. Sauter, *For the Love of Geometry*, cit., p. 191.

11 Claude Pavur, S.J. (ed.), *The Ratio Studiorum. The Official Plan for Jesuit Education*, Saint Louis, The Institute of Jesuit Sources, 2005, p. vii.

12 Ivi, p. 109; James M. Lattis, *Between Copernicus and Galileo. Christoph Clavius and the Collapse of Ptolemaic Cosmology*, Chicago and London, The University of Chicago Press, 1994, p. 32.

Clavius himself published extensively on mathematics, most notably commentaries on Euclid's *Elements* and Sacrobosco's *Sphaera*, and commended the discipline for its epistemological certainty and its utility for other fields[13]. The first edition of his commentary on the *Elements*, in that vein, praises Euclid as the portal to other mathematical sciences[14]. The dedicatory letter to the commentary's revised edition of 1607, moreover, touches upon said transcendent dimension of geometry:

> Et […] si nihil aliud, hoc sane nostrum illud studium artemque abundantissime commendabit, laudibusque accumulabit multifariis, quod Plato dicit, Deum semper γεομετρεῖν, hoc est, certissimo motu omnia metientem gubernare haec inferiora. […] Cum itaque huius summi artificis lineas non considerare, hacque pulcherrima Geometria, quae diuinitatem nobis ostendit, Deumque monstrat, non delectari, nec illius tam luculentia vestigia aspicere aut inquirere, summa & manifesta sit impietas, nemo me hercle […] vitio unquam vertet […].
>
> And […] if nothing else, then surely this will amply commend our effort and our discipline, and bestow manifold praise, namely, that Plato claims God to always *geometrize*, that is, to govern the things below by measuring everything with steadiest movement. […] Since it would therefore be an egregious and manifest impiety not to contemplate the lines of the sublime craftsman, and not to delight in this beautiful Geometry, which reveals divinity to us and shows us God, and not to behold or investigate his traces so splendid, surely nobody will ever find fault with me […][15].

Clavius thus brands regular patterns in the cosmos as a deliberate trait of divine agency – and geometry, the tool to their discovery, as a connective tissue to God as the world's creator. The Jesuit mathematician himself followed his own precept to the letter: his commentary on

13 Ivi, p. 35.

14 Christopher Clavius, *Euclidis Elementorum Libri XV*, Rome, Accoltus, 1574, see 'Ad Lectorem'.

15 Christopher Clavius, *Euclidis Elementorum Libri XV*, Frankfurt, Hoffmann, 1607, sig*4v–*5 (translation added by author).

Johannes Sacrobosco's *Sphaera*, a widely used astronomy primer, inserts numerous geometric diagrams to aid the conceptual grasp of the universe, and an additional chapter on isoperimetric figures[16].

The geometric tiles installed in Coimbra College more than a century later testify to a shared enthusiasm for Euclid. In his own days, however, Clavius faced an uphill battle convincing his brethren of the benefits of a mathematically bent curriculum. Contrary to his lobbying efforts, the *Ratio Studiorum* of 1599 eked out a minimal role for the discipline. Instead of a three-year course on mathematics suggested in a draft of 1586, the final version merely prescribed lessons on Euclid and astronomy to students of natural philosophy. In the judgment of Karp and Schubring, «[f]or their ultimate goal, the formation of faithful priests, mathematics clearly played no integral role for the Jesuits»[17]. This may have been the consequence of epistemological snobbery – natural philosophers and theologians still considered the syllogism's demonstrative powers superior to the deductive proofs used in Euclid's *Elements*[18]. Yet even then, the appeal of mathematics' practical utility provided the Jesuit enterprise with a considerable pull factor in Europe and abroad, Japan being a notable example[19]. Thus, if education was a means to an end – proselytization – then mathematics provided a means to the means.

Particularly in Portugal, where maritime exploration and training of skilled navigators was a national concern, applied mathematics posed a strong asset to Jesuit education. The tiles depicting Euclidean geometry in Coimbra College fit well within this branding programme.

16 Christopher Clavius, *In Sphaeram Ioannis de Sacro Bosco Commentarius*, Rome, Victorius Helianus, 1570, pp. 81-104.

17 Alexander Karp, Gert Schubring (eds.), *Handbook on the History of Mathematics Education*, New York, Heidelberg, Dordrecht, London, Springer, 2014, pp. 134-135.

18 James M. Lattis, *Between Copernicus and Galileo*, cit., p. 33.

19 D. Max Moerman, *The Epistemology of Vision: Buddhist versus Jesuit Cosmology in Early Modern Japan*, in Angelo Cattaneo, Alexandra Curvelo (eds.), *Interactions between Rivals: The Christian Mission and Buddhist Sects in Japan (c.1549-c.1647)*, Berlin, Peter Lang, 2021, pp. 319-357: see pp. 319-320.

Although decorated classrooms were not unusual for Jesuit colleges, mathematical tiles have to date only been found in Portugal[20]. Apart from Euclidean theorems, the tiles in Coimbra also feature schematic drawings from astronomy and hydraulics; the wall decoration in the Colégio de Santo Antão, at that time Portugal's foremost institution teaching applied mathematics, depicts personifications of nautical sciences, geometry, astronomy, and military architecture. Both cases exemplify the interplay between mathematical theory and practical application and the unity of the Jesuits' pedagogical ideal incorporating mathematics in a curriculum otherwise dedicated to theology, oratory, and philology[21].

This transmigration of geometric themes into the service of other disciplines not only shows up on Jesuit classroom walls, but also in print. The Jesuit curriculum may not have yielded to Clavius' interests all the way, but the idea of the rational universe and the spiritual dimension of science pervades elsehow. The Jesuit teaching edifice rested on Aristotle's philosophy and Thomas Aquinas' theology – in fact, it had been explicitly and doctrinally vested in these two authorities from the very beginning onwards by the Order's founder Ignatius Loyola in his *Constitutions of the Society of Jesus*[22]. As scholars have argued, Aristotle's works were not simply adopted to the letter but rather served as a framework; the content could be modified and adapted to provide new vantage points for natural philosophy[23]. This strategy also transpires in the Coimbra Commentaries, the authoritative work of Aristotelian exegesis compiled by Jesuit scholars in Coimbra over several decades and published 1593 onwards. Issued at the tail end of a century where Medieval cosmology had come under terse review, it presents a recon-

20 Henrique Leitão, Samuel Gessner, *Una tribus ratio*, cit., p.1.

21 Ivi, p. 5.

22 Luís Miguel Carolino, *Astronomy, Cosmology and Jesuit Discipline, 1540-1758*, in Ines G. Županov (ed.), *The Oxford Handbook of the Jesuits*, Oxford, Oxford University Press, 2019, pp. 670-707: see p. 673.

23 Sheila J. Rabin, *Early Modern Jesuit Science. A Historiographical Essay*, in «Journal of Jesuit Studies», vol. 1, 2014, pp. 88-104: see p. 93.

solidation of the Aristotelian knowledge system and proffers new arguments defending central tenets that had come under fire[24]. Geometry had a role to play in this context – more so than arithmetic, in fact. There are several references to Euclid in Coimbra's commentary on Aristotle's *De Coelo*, also referring to Clavius' works[25]. Yet unlike the latter, Mário Santiago de Carvalho notes, the Coimbran exposition «did not mathematize the world nor did it try to make up for it by promoting geometry» – it aimed to provide a rational framework incorporating heaven and earth within one consistent system[26]. In that sense, we do not find actual calculations or diagrams. Yet a number of geometric thought experiments appear in tandem with what de Carvalho terms a sort of «experientialism» whereby «pseudo-empirical» methods were used to substantiate Aristotelian dogma[27]. To cite one such example, the finitude of elements that make up the Earth is explained in analogy to the dissection of a geometric figure or body into component parts, which, so the Jesuit theory goes, must be in finite number[28].

This mode of argument also travelled to Japan with Pedro Gomez, who had taught in Coimbra and whose lecture notes fed into the compilation of the Coimbra Commentaries after his departure[29]. Appointed superior of the Bungo area in Japan, Gomez initiated a seminar for philosophy and theology in the college of Finai and authored a manual on the Catholic Faith in 1593, combined in a tripartite compendium with one treatise each on astronomy (*De Sphaera*) and on the soul (*De*

24 Edward Grant, *The Partial Transformation of Medieval Cosmology by Jesuits in the Sixteenth and Seventeenth Century*, in Mordechai Feingold (ed.), *Jesuit Science and the Republic of Letters*, Cambridge, MA, MIT Press, 2003, pp. 127-155: see pp. 127-129.

25 See e.g. Collegium Conimbricense, *Commentarii Collegii Conimbricensis Societatis Iesu in Quatuor Libros de Coelo Aristotelis Stagiritae*, Lyon, Giunta, 1594, pp. 7 and 450.

26 Mário Santiago de Carvalho, *The Coimbra Jesuit Aristotelian Course*, Coimbra, Coimbra University Press, 2018, pp. 89-90.

27 Ivi, cit., p. 87.

28 Collegium Conimbricense, *Commentarii Collegii Conimbricensis Societatis Iesu in Quatuor Libros de Coelo Aristotelis Stagiritae*, cit., p. 419.

29 Mário Santiago de Carvalho, *The Coimbra Jesuit Aristotelian Course*, cit., p. 17.

Anima)[30]. The *Sphaera* is not an autograph nor signed by Gomez, but the responsibility for the content is commonly ascribed to him[31]. The astronomy it presents shows some affinity to the Coimbra Commentaries insofar as it interweaves philosophical argument, theological dogma and spatial projection – akin to said «pseudo-empirical» thinking. The perhaps most daring feature shared between the *Commentaries* and Gomez' manual is the square shape of the so-called Empyrean Heaven. The Empyrean Heaven is a theological concept, not a scientific one; it denotes a realm up above where the blessed dwell in the afterlife. Although a purely imagined entity, the Empyrean found its way into scientific depictions of the Medieval cosmos, in analogy to the spheres containing planets and fixed stars[32]. Drawn as the outermost sphere in the geocentric universe, the Empyrean Heaven underwent a geometric transformation in the hands of the Coimbra Jesuits: unlike the mobile orbs carrying planets and fixed stars, they argued, it was probably square, since a square shape is more conducive to immobility as the realm of the blessed was imagined to be[33].

The Square Empyrean Heaven is not merely the visual projection of a spiritual thought (or faith) experiment. It is symptomatic for the rationalistic blending of theology and science, exemplified by the spatial reconciliation of the real and the imagined rendition of their shared sphere of interest: Heaven. In that sense, the geometric tiles in Coim-

30 See Vatican Library, Reg.lat.426; Joseph Schütte, (S.J.), *Drei Unterrichtsbücher für japanische Jesuitenprediger aus dem XVI. Jahrhundert'*, in «Archivum Historicum Societatis Iesu», vol. 8, 1939, pp. 223-256: see pp. 225-228.

31 Hiraoka Ryuji, Akihiko Watanabe, *A Jesuit Cosmological Textbook in "Christian Century" Japan: De Sphaera of Pedro Gomez (Part II)*, in «SCIAMVS», vol. 16, 2015, pp. 125-223: see pp. 127-128.

32 John North, *Astronomy and Astrology*, in David C. Lindberg, Michael H. Shank (eds.), *The Cambridge History of Science*, Cambridge, Cambridge University Press, 2013, II, pp. 456-844: see p. 456.

33 Collegium Conimbricense, *Commentarii Collegii Conimbricensis Societatis Iesu in Quatuor Libros de Coelo Aristotelis Stagiritae*, cit., p. 245; Ryuji Hiraoka, *Jesuit Cosmological Textbook in "the Christian Century" Japan: De Sphaera of Pedro Gomez (Part I)*, in «SCIAMVS», 6, 2005, pp. 99-175: see p. 112.

bra not only make a point on the certainty and utility of mathematics. They expound something more fundamental and perhaps fundamentally Christian: the belief in a rational universe which can be rationally explored by a rational mind, to the point of knocking on the door of the ineffable. The very unity created between science and theology is fuelled by the idea that the rational exploration of the universe would bring the mind closer to God. It therefore opens a vast field for logic-based proselytization, starting with Clavius' *luculenta vestigia* and rising to the heights of Coimbra's square Empyrean heaven. What this unity of science and faith achieves, moreover, is the link between logic and faith-based articles: the "true" religion, as the Christians conceived of it, can now be proven "true" through scientific argument – and defended against "false" contenders.

2.2. Mathematics in Edo Japan: Pursuit, not projection

In Japan, the Jesuits invested considerable effort in proving Buddhism and Shintoism "false", supporting their monotheistic worldview with a bedrock of scientific knowledge[34]. This attitude differed considerably to that of their host country, which had historically persisted in a multi-faith environment where Buddhism, Shintoism and later Neo-Confucianism co-existed or even blended. Unlike Christendom, Japan's spiritual landscape did not rest on a cosmogonical narrative precipitated by God's deliberate agency[35]. The *Kojiki*, for example, Japan's mytho-historical account of "Ancient things" predates the existence of the gods with spontaneous formation of matter, expressed in deliberately vague terms:

34 Charles Boxer. *The Christian Century in Japan, 1549-1650*, Manchester, Carcanet, 1993, pp. 220-221; Yoshihisa Yamamoto, *Scholasticism in Early Modern Japan*, in «Mediaevalia. Textos e Estudos», vol. 31, 2012, pp. 251-279: see p. 262.

35 Angelo Cattaneo, *Spatial and Linguistic Patterns in Early Modern Global History*, in Angelo Cattaneo, Alexandra Curvelo (eds.), *Interactions between Rivals: The Christian Mission and Buddhist Sects in Japan (c.1549-c.1647)*, Berlin, Peter Lang, 2021, pp. 277-318: see pp. 297-298.

2: When the primeval matter had congealed but breath and form had not yet appeared, there were no names and no action. Who can know its form?

3: However, when heaven and earth were first divided, the three deities became the first of all creation. The Male and Female here began, and the two spirits were the ancestors of all creation[36].

As Philippi explains, the *Kojiki*'s potpourri of myths, genealogies and tales has to be understood as an attempt to justify existing social hierarchies and to locate them within this socio-mythological structure[37]. As that, the work does not bear any cosmogonic pretences. In fact, the vagueness concerning the very beginning of it all is instructive: if we believe the *Kojiki*, the origin of the universe better be beyond human understanding. And just as the coming-to-be of the world is not presented as an orderly, willed creation, there is no single God responsible for the world's design – and no conceptual link between regularities observed in the universe and a divine masterplan.

As of the beginning of the eighteenth century, a number of Buddhist scholars did attempt to map the universe using geometric diagrams, in close combat with their rationalist rivals from Europe[38]. Yet these projections are not drawn from techniques familiar from *sangaku* nor is there a particular link between Buddhism and *wasan*. In fact, *sangaku* do not appear to be tied to a specific religion or belief; they were dedicated in Buddhist temples and Shinto shrines alike, across a whole country featuring a variety of creeds and traditions. As that, they are the collective product of a decentralized network of people varied in faith and practice; their common denominator is their pursuit of mathematics.

Some *sangaku* problems playfully reflect on applied mathematics, such as the measurement of mountain tops or the area or volume of physical objects; one problem from a *sangaku* dedicated at Manpukuji in Date, Fukushima Prefecture, even maps the sun, the moon and two

36 Donald Philippi (ed.), *Kojiki*, Tokyo, University of Tokyo Press, 1968, p. 37.

37 Ivi, p. 13.

38 D. Max Moerman, *The Epistemology of Vision*, pp. 326ff.

stars in a miniature cosmos[39]. But even though material realities may have provided attractive backdrops for calculation problems, *wasan* scholars were not primarily driven by practical concerns, and on occasion even signal a self-conscious distance thereof[40].

The following is an attempt to locate the role of *sangaku* within Edo Japan's intellectual landscape to understand whether we can speak of a spiritual dimension at all – or whether the presence of *sangaku* in temples and shrines is merely an accident of location.

The earliest mention of *sangaku* places the custom in the middle of the 17th century[41]. From the point of timing, this postdates the Jesuit presence in Japan and the official commencement of Edo Japan's isolationist policy, decreed through four *sakoku* laws issued from 1633 to 1636[42]. More poignantly, the rise of *wasan* and of *sangaku* follows in the wake of a new influx of Chinese mathematical treatises around 1600[43]. *Wasan* not only spread as a consequence of imported knowledge; as Terence Jackson argues, the Tokugawa period brought political and socioeconomic changes, the political unification of the country not the least of them, which spurred an outright information revolution «systematized in part around extensive social networks and the expanding demand for information among all classes»[44]. For one, a decree by Toyotomi Hideyoshi in 1591 officially divided society into four hierarchical classes: samurai, farmers, artisans and merchants. In the

39 Many thanks to Kansei Hirabayashi, Chief Priest of Manpukuji in Date, Fukushima, for showing me this unique problem.

40 Mark Ravina, *Wasan and the Physics That Wasn't. Mathematics in the Tokugawa Period*, in «Monumenta Nipponica», vol. 48, No. 2, 1993, 205-224: see p. 206.

41 Sujatha Ramdorai, *Interview with Hidetoshi Fukagawa*, cit., p. 34.

42 Daniele Frison, *Contextual Chronology of the Christian Mission in Japan c.1540-c.1640*, in Angelo Cattaneo, Alexandra Curvelo (eds.), *Interactions between Rivals: The Christian Mission and Buddhist Sects in Japan (c.1549-c.1647)*, Berlin, Peter Lang, 2021, pp. 46-59: see p. 58.

43 Richard Rubinger, *Private Academies of the Tokugawa Era*, Princeton (NJ), Princeton University Press, 1982, p. 183.

44 Terrence Jackson, *Network of Knowledge: Western Science and the Tokugawa Information Revolution*, Honolulu, University of Hawaii Press, 2016, p. 11.

absence of war, the samurai were reinvented as a class of noble bureaucrats with access to formal education in governmental schools. Besides that, however, two types of schools existed that were open to all social strata: temple schools (*terakoya*), and private academies (*juku* or *shijuku*), which offered a wide range of disciplines from practical skills like reading and writing to cultural pursuits like philosophy, calligraphy and martial arts[45]. By the end of the 19th century, some 80.000 *juku* are thought to have existed throughout Japan[46].

The country presumably profited from literacy trickling down the ranks. Still, it seems that Edo Japan's diversified educational landscape was less a nod to necessity than a curious peacetime dividend. In a land newly pacified, the former warrior class still functioned as the country's aristocracy. With trade burgeoning during the Edo period, however, social status and concomitant obligations were not always matched by financial security; in fact, the rising income of commoners stood in contrast to the static stipend paid to the samurai in service of the government. The rise of real wages meant many samurai families had to let go their servants, and lower-ranking nobles even took on part-time jobs to supplement their income[47]. Running a *juku* hence provided an additional source of income for well-educated but cash-strapped samurai.

A *juku's* curriculum depended on the headmaster's specialization, and the disciplinary and institutional boundaries were by no means hard and fast. Many Confucian or "National Studies" *juku* included lessons in mathematics and writing alongside their headline subjects. Granted most *terakoya* also focused on these two skills, there must have been a considerable overlap with the lower-level *juku* if we judge

45 Kristina Hmeljak Sangawa, *Confucian Learning and Literacy in Japan's Schools of the Edo Period*, in «Asian Studies», vol.5, No. 2, 2017, pp. 153-166: see pp. 155-156; Richard Rubinger, *Private Academies of the Tokugawa Era*, cit., *passim*.

46 Annick Horiuchi, *Japanese Mathematics in the Edo Period (1600-1868)*, cit., p. 20.

47 Susan B. Hanley, *Tokugawa Society: Material Culture, Standard of Living, and Life-Styles*, in John Whitney Hall, James McClain (eds.), *The Cambridge History of Japan*, Cambridge, Cambridge University Press, 1991, pp. 660-705: see p. 703.

the school by its content[48]. It was the *juku*, however, and their affiliation with renowned masters which fostered a nationwide exchange of scholars at a time when travel of people and knowledge was officially restricted[49]. Two programmes received governmental sanction, *yuureki* (遊歴)and *yuugaku* (遊学), which allowed scholars or promising students to go on educational journeys or study at schools in a different jurisdiction or fiefdom. This movement of people promoted considerable exchange of knowledge and growth of networks across Japan[50]. *Sangaku*, which typically record the problem author's educational affiliation, pay testament to the size and spread of these networks: some tablets cite a school or master at a veritable distance from the author's location[51].

The custom of *sangaku* therefore creates a visible surface for two different stimuli alive in *wasan*'s social dimension: relationships and bonds within intellectual circles, and rivalry between competing schools and scholars. Arguably, this (largely) friendly fire did more to foster *wasan* as a discourse than the ambition to solve real life problems. As that, it makes more sense to interpret *wasan* as a cultural pursuit with a social rather than a utilitarian dimension, particularly if we consider it in the context of other *juku* where more ostensibly disinterested arts like *ikebana* or calligraphy were practiced.

Revisiting *sangaku* in the context of *wasan*'s print publications provides further hints as to their origin. Annick Horiuchi credits the *Jinkoki*, a popular Japanese language manual first published 1627 by Yoshida Mitsuyoshi, with the provision of a common mathematical

48 Richard Rubinger, *Private Academies of the Tokugawa Era*, cit., p. 182.

49 Ivi, p. 15.

50 Ivi, pp. 24-26.

51 The *Shinpeki Sanpo* (神壁算法, or «Mathematics of God's Wall»), a collection of sangaku problems published in the Edo period, records *sangaku* problems dedicated by members of the Seki Takakazu school in the whole country, from Hokkaido down to Kyushu, see Rosalie Hosking, *Solving Sangaku: A Traditional Solution to a Nineteenth Century Japanese Temple Problem*, in «Journal for History of Mathematics», vol. 30, No. 2, 2017, pp 53-69: see pp. 58-59.

vocabulary and solid instructions for abacus computation – which, in her view, «very likely favored the constitution of a wide and unified community of mathematical specialists across the country»[52]. In 1641, Yoshida published a new edition *(Shinpen jinkoki*, 新編塵劫記), which poignantly introduced the practice of *idai*, or "Bequeathed questions": a set of unsolved problems in the appendix, designed to test the skills of mathematics masters. Yoshida's first set of "Bequeathed questions" duly sparked off a response in kind where mathematicians strove to publish their solutions combined with new unsolved problems[53]. *Sangaku* problems mirror this practice on wood; and incidentally, the earliest ones are thought to have been dedicated around the time when the fashion of *idai* consolidated[54]. Without a doubt, Yoshida not merely used this strategy to throw out a gauntlet, but also as a means of advertising. Often enough, *wasan* publications did not disclose the full extent of the author's knowledge so as to attract students rather than fully satisfy readers, also in the understanding that the proper place to transmit such knowledge was the classroom, from person to person[55]. *Sangaku*, likewise, do not showcase solutions but only problems, answers, and techniques – and the name of the school with the curriculum to match. Intentional or not, they quite naturally served as colourful advertising boards.

Judging from the above, it is possible to view *sangaku* as items promoting a purely secular concern with no explicit reference to divine worship, a deeper philosophical portent or spirituality. Considering them as material manifestations of *wasan* as a practice, however, allows us to revisit *sangaku* in the context of Edo Japan's intellectual landscape, at a time when Neo-Confucian thinking rapidly spread and

52 Annick Horiuchi, *The Jinkōki Phenomenon: The Story of a Longstanding Calculation Manual in Tokugawa Japan*, in «Listen, Copy, Read: Popular Learning in Early Modern Japan», Leiden and Boston, Brill, 2014, pp 253–287: see p. 266.

53 Ivi, pp. 274-275.

54 Nobuya Nakamura, Tetsuo Sekiguchi, *Sanpo-Koren & Kankai-No-Utsuwa*, cit., p. 13.

55 Annick Horiuchi, *The Jinkōki Phenomenon*, cit., pp. 272-273.

consolidated across Japanese society[56]. As it is, the Chinese mathematical tradition *wasan* hailed from bore Confucian undertones that seem to have taken root on Early Modern Japanese soil – unsurprisingly perhaps, granted that in the world of Edo *juku*, *wasan* and Confucian learning lived at close quarters. For one, mathematics was considered one of the "Six Arts" in Confucian philosophy, a curriculum of disciplines including chariot-driving, literature, music, archery, and the performance of rites besides. Education of this kind was expected to make a man not merely well-rounded, but also more virtuous[57]. The "Six Arts" are also referenced in the preface to the *Sangaku Kochi* (算学鉤致), a compilation of *sangaku* problems published in 1819 by Ishiguro Nobuyoshi, and the *Sanpo Koren* (算法琁璉), published by Tadayoshi Kobayashi[58].

Content and presentation of mathematical knowledge, moreover, suggest an interplay with Confucian and Daoist ideas from Chinese roots onwards. Commentaries on the *Jiuzhang suanshu*, for example, a Chinese mathematics book of major influence in early Edo Japan, feature numerous quotations from the classics of Confucianism, such as the *Analects*, and the *I ching*. *Sangaku* as a medium, moreover, exemplify strategies of Confucian pedagogy, which stressed the stimulation of effort and deeper understanding through partial presentation of knowledge. With a healthy distrust towards discursive, exclusively language-based reasoning, «non-linguistic modes of communication such as those based on computations or figurative techniques» were considered preferable[59]. Both are regular features of *sangaku* problems. Some

56 Peter Nosco, *Introduction: Neo-Confucianism and Tokugawa Discourse*, in Peter Nosco (ed.), *Confucianism and Tokugawa Culture*, Princeton, NJ, Princeton University Press, 1989, pp 3-26: see p. 8.

57 Marina Wong, *A Comparison between the Philosophies of Confucius and Plato as Applied to Music Education*, in «The Journal of Aesthetic Education», 32.3 (1998), 109-112: see p. 110.

58 Tadayoshi Ishiguro (石黒信由), *Sangaku Kochi*(筭學鉤致), 加州, 鹽屋與三兵衛, 1819; Nobuya Nakamura, Tetsuo Sekiguchi, cit., p. 42.

59 Jean-Claude Martzloff, *Geometry in Chinese Mathematics*, cit., p. 1008.

wasan problems even reference Confucian thought quite literally: the *Kankai-no-Utsuwa-Zusetsu* by Tadayoshi Kobayashi, for example, solicits the calculation of the volume of *Kankai-no-Utsuwa*, a cup with a rounded bottom which tips over when filled beyond measure, serving as a metaphor for moderation[60].

Tadayoshi's book signals the vicinity of Confucian discourse and mathematical practice. There is, moreover, one example which bears direct witness to the role of mathematics for the kind of self-development central to Neo-Confucian thinking. In an essay known as "One Chapter on a Theory of Proper Character", the famous Edo mathematician Takebe Katahiro recounts his dawning recognition that the truthful pursuit of mathematics depends on the acceptance of one's own character:

> A person at one with the Way of mathematics and active in its practices is honest and true. The truth of what it means to be at one with the Way is not something that can be reasoned about. But in our training for this truth that is not to be reasoned about, one rule that can be affirmed is the need to follow the character one was born with[61].

As Tsukane Ogawa argues, Takebe's pronouncements have to be understood against the Neo-Confucian backdrop of his own time[62]. In the context of his argument, Takebe describes how he came to accept his own character and develop his own style of mathematics, emancipating from the techniques used by his famous master, Seki Takakazu. In the process, a reciprocal relationship between the "Way of Mathematics" and the ideal of "Know Thyself" develops which, in the opinion

60 Nobuya Nakamura, Tetsuo Sekiguchi, *Sanpo-Koren & Kankai-No-Utsuwa*, cit., pp. 58-61.

61 Tsukane Ogawa, *Takebe Katahiro – A Man of His Times: A Survey of His Life and Mathematical Thought*, in Tsukane Ogawa, Mitsuo Morimoto (eds.), *Mathematics of Takebe Katahiro and History of Mathematics in East Asia*, Tokyo, Mathematical Society of Japan, 2018, pp. 3-28: see pp. 24-26.

62 Ivi, *passim*.

of Takebe, ideally leads a person to be «honest and true». Mathematical practice, self-development and moral betterment are cast in direct correlation with each other.

Strictly speaking, Neo-Confucianism, although often blended with Shinto and Buddhism in Edo Japan, is not a religious but a moral, a philosophical framework. Yet it is a framework extending across religious and secular concerns alike, interweaving the social and the political sphere, alongside ritual practice and self-cultivation[63]. In Mary Evelyn Tucker's interpretation, if we generally accept religion as a means and impulse for self-transformation, through a deeper understanding – and understanding of the limits thereof – of phenomenal reality, then «Confucianism can certainly be regarded as religious in the sense that the primary activity of Confucians is the establishing of moral reflection and spiritual awareness within the changes of cosmological processes»[64]. The Confucian context does not speak of a personal God as Christianity would; heaven is not imagined as a pseudo-physical realm but as a creative principle sustaining the order of the cosmos and immanent within human nature. Thus, participation in the cosmological order of things does not require divine intervention or salvation, but the will and effort to understand and actualize one's own true nature[65].

If we consider Takebe's confession in this context, then *wasan* as a practice demonstrably carries the potential for spiritual experience within, in the Confucian sense. *Sangaku* as material manifestations of *wasan* signal mastery and self-development of the individual, and emulation and competition amongst a likeminded community. A spiritual experience may not be the prime intention nor are *sangaku* a deliberate product of Edo Japan's Neo-Confucian makeover. Yet the ideological backdrop does help to appreciate the revaluation of mathematics for the sake of practice rather than outcome, and its potential for aiding personal growth – and

63 Mary Evelyn Tucker, *Religious Dimensions of Confucianism: Cosmology and Cultivation*, in «Philosophy East and West», vol. 48, No. 1, 1998, pp. 5-45: see p. 21.

64 Ivi, p. 14.

65 Ivi, pp.14-15.

why, although perhaps not in reference to their location in temples and shrines, *sangaku* could bear a spiritual implication within.

3. Conclusion

If we return to the initial question concerning spiritual connotations of geometry on display in sacred spaces in Europe and Japan, the most immediate observation may also be the most telling in that regard: the Euclidean tiles portray theorems – stable truths. *Sangaku* do not purport facts – they pose questions. Seen as a medium of communication, either type of object symbolizes the heuristics particular to its mathematical discourse, and by extension the epistemology inherent in its spiritual backdrop.

Europe prized Euclid's system for its stability, universality, and utility manifest through practical application, including proselytization. Axiomatic thinking is shared by theological and scientific discourse alike, and perhaps it is the over-arching quest for certainty that renders geometry fertile for mapping the scientific-theological universe we encounter in Jesuit discourse. The utility Western mathematicians hailed, *wasan* scholars more or less deliberately distanced themselves from: *wasan* generally styled itself a discipline apart from practical thinking, and all the nobler thereby. In that sense, *sangaku* do not pay homage to the orderly universe and man's penetration thereof, like the Euclidean tiles would – *sangaku* signal human achievement, prowess more than progress, endogenous transformation more than exogenous truth, and a continuous interplay within the social fabric besides. Spiritually speaking, therefore, Western and Eastern mathematics point in opposite directions, seeking the order of the cosmos in the external world or within human nature, respectively. What is more so, either realm seems to identify different moving targets: for Christian morality, understanding the world as a stable framework of rules and regularities warrants character adjustment; if we believe Takebe as spokesman for Confucian mathematics, the character is stable, but the mathematics can be adjusted.

All that said, a poignant similarity remains: the joy that mathematics can render to those who master it. Christopher Clavius praised the

pleasure we perceive when gaining clarity through mathematics[66]. Takebe, too, speaks of a peace of mind palpable when embarked on the "Way" of mathematics[67]. Perhaps mathematics is not the universal language it is sometimes made out to be; but the psychological experience of a calculation clean-cut may be a feeling shared across the globe.

Acknowledgments

The Sangaku Archive Project is generously supported by a 2024 KAKENHI Early Career Researchers Grant (Project number 24K15964). Many thanks to Tianle Yang for deciphering and translating *sangaku* and related content and to the reviewers of this article for their help in improving this piece. I would also like to thank Yamatotakeru Shrine in Koriyama, Fukushima, for allowing me to include a picture of their *sangaku*. My warmest gratitude to Masako Watanabe, Chief Priestess of Kashima Daijingu in Koriyama, Fukushima, for her tireless efforts and kindness acting as intermediary and guide in the Miharu area.

Riassunto Durante il periodo di isolamento, il Giappone dell'epoca Edo vide l'emergere di *wasan* come discorso matematico idiosincratico. Il presente studio giustappone *wasan* come fenomeno culturale alla ricezione della geometria euclidea da parte dei Gesuiti, considerando le implicazioni epistemologiche della matematica nel contesto della spiritualità nel Giappone e nell'Europa della prima età moderna.

Abstract During its period of seclusion, Edo Japan witnessed the emergence of *wasan*, an idiosyncratic mathematical discourse. Following study juxtaposes *wasan* as a cultural phenomenon with the Jesuits' reception of Euclidean geometry, to then consider the epistemological implication of mathematics within the context of spirituality in Edo Japan and early modern Europe.

66 Christopher Clavius, *Euclidis Elementorum Libri XV*, 1574, sig a3.

67 Tsukane Ogawa, *Takebe Katahiro*, cit., pp. 24-25.